Introduction to Logic:

Argumentation and Interpretation

Vysoká škola mezinárodních a veřejných vztahů

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Introduction to Logic: Argumentation and Interpretation

Annotation

The course offers an overview of topics in logic, communication, reasoning, interpretation and summary of their practical use in communication. It provides basic orientation in terminology of linguistic research and communication, persuasion and communication strategies, understanding the logic games, exercises and tasks, and offers the opportunity to learn the reasoning applied in various situations. The aim is that students not only get familiar with lectures, but also acquire the means of communication and argumentation through exercises and online tests.

Topics

1. Brief history of Logic and its place in science
2. Analysis of complex propositions using truth tables
3. The subject-predicate logic – Aristotelian square
4. Definitions and Terminology
5. Polysemy, synonymy, homonymy, antonymy
6. Analysis of faulty arguments
7. Interpretation – rules and approaches
8. Analysis of concrete dialogue

http://mediaanthropology.webnode.cz/kurzy/introduction-to-logic/

Propositional logic

Propositional logic is based on the assumption that compound statement is true or false depending on what is the "nature" of the connective (copula) that connects the statements. Propositional logic analyzes the sentence up to the level of elementary statements. The structure of these elementary statements is not studied further. A statement is already from ancient antiquity understood the sentence that is true or false, ie. has the truth value. Truth values are two:

- true (1)
- false (0)

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Úsudky. Testy studijních předpokladů. [online] Brno, Masarykova univerzita 1996-2013. Online: http://www.muni.cz/tsp/usudky?lang=cs

Propositional logic

Propositional logic and truth values can be applied to some natural language sentences. It should be omitted:

- sentences with no truth value (What is the time? Go for newspaper!)
- sentences, which can not be clearly assigned a truth value (I am Czech. – may be true or false, and can not be decided without the context)
- sentences whose truth value is undecidable (Current Czech President is Vaclav Havel. – depending on the time when the sentence is spoken)

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

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Propositional logic

Statements are divided into **simple** and **complex**. A simple statement is such statement that has no smaller part being a statement. Complex statement, however, has its own parts – the statements.

Propositional logic examines the structure of these complex statements (propositions) in the sense that it examines comprising of simple statements into complex through **logical connections**. It is therefore a theory of logical connections. It maintains the **principle of compositionality,** according to which the **truth value of complex statement is intended by the truth values of its components and by the nature of the combination of these components** (ie. The logical nature of the connections).

Source:*E-vyuka pro logiku.* [online] Dostupné elektronicky: http://snug.ic.cz/index.htm#

Propositional logic – Examples of complex statements:

In Prague there is raining and in Brno there is sunny.

| | |

elementary statement connective el. statement

It is not true that it is raining in Prague.

| |

conn. elementary statement

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Úsudky. Testy studijních předpokladů. [online] Brno, Masarykova univerzita 1996-2013. Online: http://www.muni.cz/tsp/usudky?lang=cs

Propositional logic – Terminology:

In general terms, a **calculus** is a formal system that consists of:

- a set of syntactic expressions (**well-formed formulas**),
- a distinguished subset of these expressions (**axioms**),
- a set of formal rules that define a specific binary relation, intended to be interpreted to be logical equivalence, on the space of expressions.

When the formal system is intended to be a **logical system**, the expressions are meant to be interpreted to be statements, and the rules, known to be inference rules, are typically intended to be truth-preserving. In this setting, the rules (which may include axioms) can then be used to derive ("infer") formulas representing true statements from given formulas representing true statements.

Source: **https://en.wikipedia.org/wiki/Propositional_calculus**

Propositional logic – Terminology:

The set of axioms may be empty, a nonempty finite set, a countably infinite set, or be given by axiom schemata.

A **formal grammar** recursively defines the expressions and well-formed formulas of the language. In addition a **semantics** may be given which defines truth and valuations (or interpretations).

The language of a propositional calculus consists of:

- a set of primitive symbols, variously referred to be atomic formulas, placeholders, proposition letters, or variables, and
- a set of operator symbols, variously interpreted to be logical operators or logical connectives.

Source: **https://en.wikipedia.org/wiki/Propositional_calculus**

Propositional logic – Terminology:

A **well-formed formula** is any atomic formula, or any formula that can be **built up from atomic formulas** by means of **operator symbols** according to the **rules of the grammar**.

Mathematicians sometimes distinguish between *propositional constants, propositional variables*, and *schemata*. **Propositional constants** represent some particular proposition, while **propositional variables** range over the set of all atomic propositions. **Schemata**, however, range over all propositions.

It is common to represent propositional constants by A, B, and C, propositional variables by P, Q, and R, and schematic letters are often Greek letters, most often ϕ (phi), ψ (psi), and χ (chi).

Propositional logic :

Simple statements are comprised in a complex statement by logical connections or so called **sentence operators**.

A simple statement:

- carries just one piece of information
- Information: what the sentence is talking about (the subject of the sentence) and what the sentence says about the subject (predicate of the sentence)

Complex statement = (simple statement) (operator) (simple statement)

For example, the question of where Peter was on Tuesday can be answered:

(Peter studied mathematics) or (Peter went to the cinema).

(Peter studied mathematics and physics) or (Peter went to the cinema).

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic – Examples of complex statements:

If we know the truth value of simple statements *Peter studied mathematics, Peter went to the cinema*, we can determine the truth value of complex statement using a sentence operator (logical conjunction) *or*.

In propositional logic, we do not care about the truth value of a particular simple statement, but about **propositional form**. It's a term that itself has no truth value, because instead of concrete simple statements it contains variables.

propositinal forms

p or q

are written as (p v q)

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

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Úsudky. Testy studijních předpokladů. [online] Brno, Masarykova univerzita 1996-2013. Onliney: http://www.muni.cz/tsp/usudky?lang=cs

Propositional logic – Examples of complex statements:

- where p, q are variables for which it is possible to substitute specific statements and assign complex statements truth value based on the knowledge of the truth-values of simple statements and knowledge of the sentence operator.

In fact we have created a function that assigns truth-values to simple statements the same as is that of the whole (complex statement).

- F (p, q) = {true, false}

The language of propositional logic must contain symbols representing all elementary statements (ie. the propositional symbols, variables, that will be either true, or false), symbols for logical connectives and any other auxiliary symbols.

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

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Propositional logic

Grammar recursively defines propositional logic formulas:

(1) Propositional symbols are formulas. /base of the definition/

(2) If the statements A, B are formulas, then the expressions (¬A), (A disjunction B), (A Conjunction B), (A Implication B), (A ≡ B) are formulas as well. /induction/

(3) There are no other formulas in propositional logic than listed under (1), (2). /lock of the definition/

Formulas constructed under paragraph (1) are called the elementary /atomic, primitive/ formulas and formulas constructed under (2) are called complex formulas. Formulas A and B are the immediate subformulas. That is the maximum number of nested, into each other embeded, pairs shows (hierarchical) order of the formula.

Source: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

Introduction to Logic: Argumentation and Interpretation

Propositional logic

1. The symbols A, B used in the induction step are not formulas (do not occur as symbols in the alphabet), but metasymbols used to indicate the formulas.

2. The use of brackets in writing of formulas can be limited by following conventions:

The complex formula of the highest order does not need to be in brackets.

In case that the priority of evaluation is not decided by the brackets nor by the priority scale, we evaluate the formula from left to right. For example the formula p Þ q Þ r Þ s is evaluated as if it was written ((pÞq)Þr Þ s.

In multimember conjunctions and disjunctions the brackets are unnecessary. Eg. instead of (p ˅ q) ˅ r and p ˅ (q ˅ r) we can only write p ˅ q ˅ r. This convention is related to the previous convention (the order of evaluation does not matter and thus we can evaluate from left to right).

Source: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

Propositional logic – Examples of complex statements:

*Consider the example of two simple statements **p** and **q**. Each of them is true or false. Combinations of mutual truth and falsity are four. E.g. Czech (grammar) conjunction "and" (or. "while") is in propositional logic understood as **conjunction** (usually labeled "Λ").*

Connection of the statement P with the statement q, ie. complex statement "p and q together„, is true only for the combination of the truth of p and q, where both are true statements; in all three remaining cases, the statement "p and q together" is false. Conjunctions, etc. are called propositional conjunctions.

So let us remember that there are (Czech) grammar conjunctions like „while" (a zároveň) "if, then" (jestliže, pak) which are in logic symbolized by propositional (propositional-logic) conjuntions.

Source:*Úsudky. Testy studijních předpokladů.* [online] Brno, Masarykova univerzita 1996-2013. Online: http://www.muni.cz/tsp/usudky?lang=cs

Propositional logic

The principle of extensionality shows that in propositional logic we are limited to statements whose truth value can be determined only from the truth values of their parts (complex statements).

Examples:

V1: The road is wet and the sky is blue.

V2: The road is wet and the grass is green.

V1: Prague is the capital of the Czech Republic and Bratislava is the capital of the Slovak Republic.

V2: Prague is the capital of the Czech Republic and Warsaw is the capital of Poland.

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

3 principles characterizing the system of classical propositional logic:

Principle two values: There are only two truth values {true, false}

Principle of clarity: Every statement has exactly one truth value.

Principle of extensionality: Statements are interchangeable if they have the same truth value.

Sometimes the principle of two values and principle of clarity is called the principle of **contradiction exclusion** (not possible for one and the same statement to be both true and false), or the **law of exclusion of the third** (statement is either true, or false).

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

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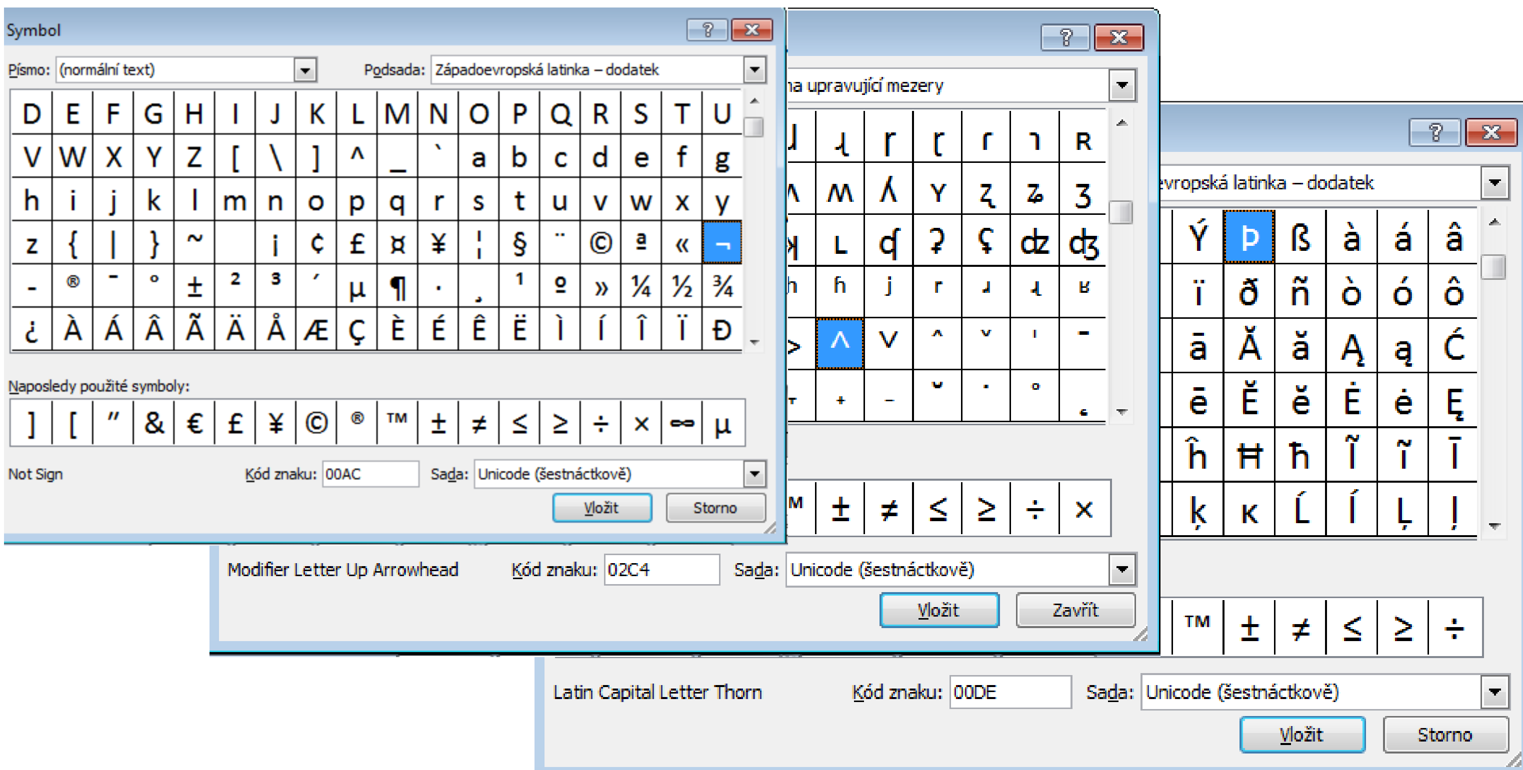
Propositional logic

Symbols ¬ ⌐ ^ ˅ Þ ≡ mark relations of

Negation, Disjunction, Conjunction, Implication and Equivalence



Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

The language of propositional logic is the set of all formulas of propositional logic. Symbolics is not uniform and standardized. The following table sets forth alternative symbols:

Symbol for connective   Alternative symbol

$\wedge$	&
$\supset$	$\rightarrow$, $\Rightarrow$
$\equiv$	$\leftrightarrow$, $\Leftrightarrow$

Source: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

Symbols ¬ ⌐ ^ ˅ Þ ≡ mark relations of

Negation, Conjunction, Disjunction, Implication and Equivalence

0	0	1	0	0	1	1
0	1	1	0	1	1	0
1	0	0	0	1	0	0
1	1	0	1	1	1	1

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.
E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Analyze the examples:

Peter´s manager:: I appreciated the performance of employee by a reward and I shifted him to a higher position.

Sdělení vedoucího Petra: Ocenil jsem výkon pracovníka odměnou a přeřadil jsem pracovníka na vyšší pozici.

If Prague is the capital of the Czech Republic, the Prague seat of government.

Jestliže je Praha hlavní město ČR, pak v Praze sídlí vláda.

Father asked whether I will stay home or go with him.

Otec se zeptal, zda zůstanu doma nebo zda půjdu s ním.

Napoleon dictated or walked.

Napoleon diktoval nebo se procházel.

If you´ll lie to me, I´ll give you a slap.

Dám ti facku, když mě oklameš.

Propositional logic

The **negation** (¬) corresponds to the phrase "*it is not true that ...*"

It is unary connective, doesn´t connect two statements (operation withn the proposition).

Example:

"*It is not true that Prague is a city*"

It is a function of one variable:

F (p) → {true, false}

Logical form: **¬** p

p: Tree is an organism. (proposition)

¬ p: The tree is not an organism.

It is not true that the tree is an organism. (complex statement)

variable	value
p	¬ p
1	0
0	1

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

The **conjunction** (**^**) corresponds to the conjunction "*and*"

Not every "and" in the natural language can be analyzed as conjunction, eg .: "Apples and pears mixed up." "I came home and turned on the heating."

It's binary, commutative connective.

Examples: "*Prague is the capital city of the Czech Republic and Prague is the seat of the Czech president*."

"*Prague is the capital of the Czech Republic and 2 + 3 = 5.*"

"Petr attends the secondary school, and (while) Petr studies mathematics."

It is a function of two variables:

F (p, q) → {true, false}

variable	variable	value
p	q	p ^ q
1	1	1
1	0	0
0	1	0
0	0	0

Logical form: **p ^ q**

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

The **disjunction** (˅) corresponds to the conjunction "*or*"

It's binary, commutative connective.

It is a function of two variables:

F (p, q) → {true, false} Logical form: **p ˅ q**

variable	variable	value
p	q	p ˅ q
1	1	1
1	0	1
0	1	1
0	0	0

Examples:

"Cars have front or rear wheel drive."

"Director Petr Novák manages in

a directive manner or uses a delegation."

BUT*:*

"This man is married or unmarried" → ¬ (p ≡ q)

"Father asked whether I stay at home or go with him" → ¬ (p ≡ q)

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

The implication (**Þ**) corresponds to phrases *"if, then"*, *"if, so"*

It is the only one binary connective that is not commutative and we call the first term ***antecedent***, the second ***consequent***. Implications does not assume a link between the antecedent´s and consequent´s content, therefore we sometimes call it ***material implication***.

It is a function of two variables:

F (p, q) → {true, false} Logical form: **p Þ q**

Examples:

"If 1 + 1 = 2, then iron the metal"

"If Peter was at home on Monday night, then he studied mathematics."

variable	variable	value
p	Q	p Þ q
1	1	1
1	0	0
0	1	1
0	0	1

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.
E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

The **equivalence** (≡) corresponds to phrases *„just when" "if and only if", etc.,* **But not** *"if"* (this is implication)

It's binary, commutative connective.

It is a function of two variables:

F (p, q) → {true, false} Logical form: **p ≡ q**

proměnná	proměnná	hodnota funkce
p	q	p ≡ q
1	1	1
1	0	0
0	1	0
0	0	1

Examples:

“Student performance is evaluated with the best mark if and only if the student has mastered the basics of logic.“

a) *"I'll slap you when you lie to me."* **→ lie Þ slap**

b) *"I'll slap you if and only if you lie to me."* **→ lie ≡ slap**

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

Conjunction and disjunction are opposite operators. While the conjunction is true if and only if all its parts are true, the disjunction is false if and only if all its parts are false. This relation is called **duality**.

Expressing conjunctive relationship using the disjunctive:

$$(p \wedge q) \rightarrow \neg(\neg p \vee \neg q)$$

Expressing disjunctive relationship using the conjunctive:

$$(p \vee q) \rightarrow \neg(\neg p \wedge \neg q)$$

This relation between conjunction and disjunction is called **De Morgan's law**.

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.
E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic

Expressing the equivalence using implication and conjunction:

(p ⇔ q) ≡ (p Þ q) ^ (q Þ p)

Formulae p, q, ¬p, ¬ q, p ∧ q, ¬ p ∨ ¬ q, ¬ (p ∧ q) , (¬ p ∨ ¬ q) ≡ ¬(p ∧ q) are achievable. E.g. formula ¬ (p ∧ q) is true (has the truth value of 1) for the evaluation 0,1 for propositional symbols p, q. Also evaluation (1.0), (0.0) are its models, but not (1,1).

The statement (¬p ∨ ¬ q) ≡ ¬ (p ∧ q) is **tautology**. For all possible evaluation of 0,0; 0,1; 1,0; 1,1 for propositional symbols (p, q) is true.

The statement ¬ [¬p ∨ ¬ q) ≡ ¬ (p ∧ q)] is **contradiction**. There is no evaluation for propositional symbols (p, q) for which it would be true.

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.
E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic – Tautologies and contradictions

Definition: A propositional expression is a **tautology** if and only if for all possible assignments of truth values to its variables its truth value is T.

Example: $P \vee \neg P$

If two expressions P and Q are equivalent, i.e. $P \equiv Q$, then the statement is a tautology.

Definition: A propositional expression is a **contradiction** if and only if for all possible assignments of truth values to its variables its truth value is F.

Example: $P \wedge \neg P$

Source: http://faculty.simpson.edu/lydia.sinapova/www/cmsc180/LN180_Johnsonbaugh-07/Overview_logic.htm#Tautologies

Propositional logic – Modus Ponens and Modus Tollens

Modus ponens (method of affirming)

(1) If P then Q

(2) P

(3) Therefore Q

Example:

If it is Sunday we go fishing.

It is Sunday.

Therefore we go fishing.

Source: **http://faculty.simpson.edu/lydia.sinapova/www/cmsc180/LN180_Johnsonbaugh-07/Overview_logic.htm#Tautologies**

Propositional logic – Modus Ponens and Modus Tollens

Modus Tollens (method of denying)

(1) If P then Q

(2) ¬ Q

(3) Therefore ¬ P

Example:

If it is Sunday we go fishing.

We do not go fishing.

Therefore it is not Sunday.

Modus ponens uses implication, modus tollens uses the contrapositive of the implication.

Source: **http://faculty.simpson.edu/lydia.sinapova/www/cmsc180/LN180_Johnsonbaugh-07/Overview_logic.htm#Tautologies**

Propositional logic – Examples of invalid arguments:

Inverse error:

If it is Sunday we go fishing.

It is not Sunday.

Therefore we do not go fishing.

Converse error:

If it is Sunday we go fishing.

We go fishing.

Therefore it is Sunday.

Source: **http://faculty.simpson.edu/lydia.sinapova/www/cmsc180/LN180_Johnsonbaugh-07/Overview_logic.htm#Tautologies**

Propositional logic in practice

1. We put the sentence in natural language to logical formulas, we find the appropriate logical form

Peter´s manager: *I appreciated the performance of the employee and I shifted him to a higher position.*

p: Peter´s manager appreciated his performance

q: Peter´s manager shifted him to a higher position.

p ? q

2. At the level of formulas we use the rules of logic to we find the truth value of complex propositions.

3. We interpret this information using natural language, in relation to the original content.

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Propositional logic in practice

Complex statement is true if and only if both situations actually occurred, ie. *the performance was appreciated* and the Peter *was shifted to a higher position*. If *the performance was not appreciated* and Peter *was shifted to a higher position*, manager´s statement is a false statement.

And if *the performance was appreciated* but Peter was *not shifted to a higher position*, manager´s statement is false as well.

variable	variable	value
p	q	p ^ q
1	1	1
1	0	0
0	1	0
0	0	0

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Propositional logic in practice - Analysis of argument´s correctness

1. **Rewrite the argument using the symbolism of propositional logic** (abstract from the language elements that are not from the perspective of the essential correctness of the argument)
2. **Determine whether the conclusion results from the premises** (ie. Whether it is excluded that the premises are true and the conclusion false).

Example:

Decide on argument´s correctness

Premise: *If Prague is the capital of the Czech Republic, Prague is the seat of the government.*

Premise: *Prague is the capital city of the Czech Republic.*

Conclusion: *Prague is the seat of the government.*

Solution: Step 1 → Final form of argument: p Þ q

upraveno podle: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Propositional logic in practice - Analysis of argument´s correctness

Step 2 → Truth values

Table of logical function implication			Form of the argument		
P	q	p Þ q	Premise	Premise	Conclusion
1	1	1	p Þ q	p	q
1	0	0	1	1	1
0	1	1	0	1	0
0	0	1	1	0	1
			1	0	0

We have identified all possible options, the case (1,1,0) does not occur in the solution, so we have excluded the possibility that the premises are true and the conclusion is false.

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Propositional logic in practice - Analysis of argument´s correctness

Each evaluation of propositional symbols contained in formula A for which the truth value of the function equal to 1 is called a model of this formula.

We will find out whether the set of formulas

M = {p **Þ** r, q **Þ** r, p **˅** q} *is achievable:*

A formula of propositional logic is achievable where it is true for some evaluation or there is at least one model of the formula A.

p	q	r	p **Þ** r	q **Þ** r	p **˅** q
1	1	**1**	**1**	**1**	**1**
1	1	0	0	0	1
1	0	1	1	1	1
1	0	0	0	1	1
0	1	1	1	1	1
0	1	0	1	0	1
0	0	1	1	1	0
0	0	0	1	1	0

The given set M is achievable and its corresponding evaluation models are the 1st, 3rd and 5th row. Furthermore, the table shows that the set M logically follows the formula r. For each model of this set r is true. So (brackets are not necessary): **p Þ r, q Þ r, p ˅ q |= r**

Source: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

The language of Propositional logic

The introduction of two logical connectors (negation, implication) is sufficient because all other logical coupling of propositional logic can be expressed by negation and implication.

Definition 1: (conjunction)

$$(A \wedge B) \equiv \neg (A \rightarrow \neg B)$$

Definition 2: (disjunction)

$$(A \vee B) \equiv (\neg A \rightarrow B)$$

Definition 3: (equivalence)

$$(A \Leftrightarrow B) \equiv ((A \rightarrow B) \wedge (B \rightarrow A))$$

This is the basic syntax of the language of propositional logic.

Source: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela. *Etika a logika v komunikaci.* Praha: UJAK, 2007.

The language of Propositional logic – Semantics

We need to relate the created formulas to truth values (because we want to use the language for the analysis of the entailment). We define a function called Truth valuation (interpretation), that assigns every single correctly formed formula a truth value (0,1).

Truth valuation is satisfied by formula A, if formula A in this evaluation is true. I (A) = 1

For all the correctly formed formulas A, B apply: (rules)

a) **I(¬ A)=1** if and only if **I(A)=0**.

b) **I(A → B)=1** if and only if **I(A)=0** or **I(B)=1**.

Souces: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela. *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Propositional logic – important tautologies:

Tautology with single propositional symbol: $|= p \equiv p$

$|= p \vee p$	exclusion of the middle
$|= \neg (p \wedge \neg p)$	the law of contradiction
$|= p \equiv \neg \neg p$	law of double negation

Algebraic laws:

$|= (p \vee q) \equiv (q \vee p)$	commutative law for $\vee$
$|= (p \wedge q) \equiv (q \wedge p)$	commutative law for $\wedge$
$|= (p \equiv q) \equiv (q \equiv p)$	commutative law for $\equiv$
$|= (p \vee q) \vee r \equiv p \vee (q \vee r)$	associative law for $\vee$
$|= (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	associative law for $\wedge$
$|= ((p \equiv q) \equiv r) \equiv (p \equiv (q \equiv r))$	associative law for $\equiv$
$|= (p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$	distributive law for $\wedge, \vee$
$|= (p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$	distributive law for $\vee, \wedge$

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Propositional logic – important laws:

Commutative laws:

$P \vee Q \equiv Q \vee P$

$P \wedge Q \equiv Q \wedge P$

Associative laws:

$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$

$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$

Distributive laws:

$(P \vee Q) \wedge (P \vee R) \equiv P \vee (Q \wedge R)$

$(P \wedge Q) \vee (P \wedge R) \equiv P \wedge (Q \vee R)$

Source:
http://faculty.simpson.edu/lydia.sinapova/www/cmsc180/LN180_Johnsonbaugh-07/Overview_logic.htm#Tautologies

Propositional logic – important laws:

Identity:

$P \lor F \equiv P$, $P \land T \equiv P$

Negation:

$P \lor \neg P \equiv T$ (excluded middle)

$P \land \neg P \equiv F$ (contradiction)

Double negation:

$\neg(\neg P) \equiv P$

Idempotent laws:

$P \lor P \equiv P$

$P \land P \equiv P$

Source: **http://faculty.simpson.edu/lydia.sinapova/www/cmsc180/LN180_Johnsonbaugh-07/Overview_logic.htm#Tautologies**

Propositional logic – important laws:

De Morgan's Laws:

$\neg (P \lor Q) \equiv \neg P \land \neg Q$

$\neg (P \land Q) \equiv \neg P \lor \neg Q$

Universal bound laws (Domination):

$P \lor T \equiv T$

$P \land F \equiv F$

Absorption Laws:

$P \lor (P \land Q) \equiv P$

$P \land (P \lor Q) \equiv P$

Negation of T and F: $\neg T \equiv F$, $\neg F \equiv T$

Source: http://faculty.simpson.edu/lydia.sinapova/www/cmsc180/LN180_Johnsonbaugh-07/Overview_logic.htm#Tautologies

Tasks:

For the proposition
"Peter is lying and stealing."

specify its negation:

a) "Peter is lying, but not stealing."
b) "Peter is not lying and stealing."
c) "When Peter is not lying, he is not stealing."
d) "Peter is not lying or stealing."
e) "Either it is true that Peter is lying, or it is not true that Peter is stealing."

Source: Tests - Study assumptions and logic. Prague, Fragment, 2007, p. 83.

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a) "Peter is lying, but not stealing."
b) "Peter is not lying and stealing."
c) "When Peter is not lying, he is not stealing."
d) "Peter is not lying, nor stealing."
e) "Either it is true that Peter is lying, or it is not true that Peter is stealing."

p	q	task	a)	b)	c)	d)	e)
1	1	1	0	1	1	0	1
1	0	0	1	1	1	1	0
0	1	0	0	0	0	1	0
0	0	0	0	1	1	1	1

Tasks:

The proposition
"Eva can ski or skate."

can be considered equivalent with the proposition:

a) "If Eva can not skate, then she can ski."
b) "If Eve can ski, she can not skate."
c) "If Eva can not skate, she can not ski."
d) "Eve can ski only if she can not skate."
e) "If Eve can ski, she can skate."

Source: Tests - Study assumptions and logic. Prague, Fragment, 2007, p. 192.

a) "If Eva can not skate, then she can ski."
b) "If Eve can ski, she can not skate."
c) "If Eva can not skate, she can not ski."
d) "Eve can ski only if she can not skate."
e) "If Eve can ski, she can skate."

p	q	task	a)	b)	c)	d)	e)
1	1	1	1				
1	0	1	1				
0	1	1	1				
0	0	0	0				

The proposition

$\neg (p \land q)$

can be considered equivalent with the proposition:

a) $(p \lor q)$

b) $(\neg q) \lor (\neg p)$

c) $p \land q$

d) $(\neg p) \land (\neg q)$

e) none of the options

p	q	task	a)	b)	c)	d)	e)
1	1	1	1				
1	0	1	1				
0	1	1	1				
0	0	0	0				

Thank you for your attention!

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